Mathematics Lecture

FLIPPING COINS, DRUNKEN WALKS, AND A SIMPLE PROOF OF WALLIS' PRODUCT FORMULA FOR π

Erik Insko, Ph. D.

Location: AB 7 Room 220

Friday, October 12, 10:30–11:30am.

We discuss the ties between the combinatorics of coin flipping problems, drunken walks, and the elementary geometric proof of Wallis' product formula for π due to Johan Wästlund. This talk is motivated by a lecture given by Donald Knuth at Bowdoin College, Wästlund's article in The American Mathematical Monthly, and a personal correspondence with aBa Mbirika.

The following excerpt from Wästlund's article explains the formula:

> In 1655, John Wallis wrote down the celebrated formula
>
> $$\frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdots=\frac{\pi}{2}.$$
>
> Most textbook proofs of this formula rely on evaluation of some definite integral like
>
> $$\int_0^{\pi/2}(\sin x)^n\,dx$$
>
> by repeated partial integration. The topic is usually reserved for more advanced calculus courses.

In this talk we will show that this formula can be derived using only the mathematics taught in elementary school: basic algebra, the Pythagorean theorem, and the formula for the area of a circle.